TECHNICAL ANALYSIS – CIRCUIT PARAMETER DETERMINATION WITH OPTOCOUPLER

Think Perfect

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1. Introduction

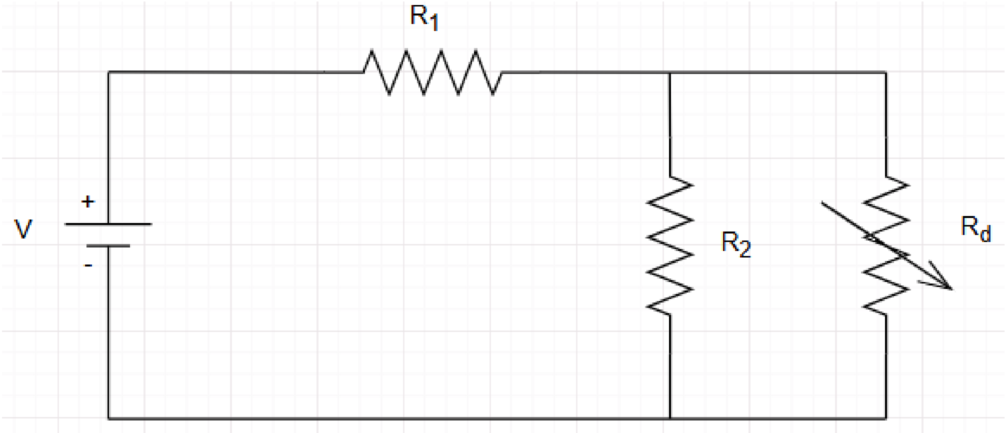
This study presents a detailed analysis of a resistive circuit incorporating an optocoupler diode (R_d). The objective is to derive the values of resistors R_1, R_2 and the dynamic resistance of the optocoupler (R_d) using experimental voltage and current measurements.

The analysis employs Kirchhoff's Voltage Law (KVL) and Ohm's Law to derive and solve a system of equations. The nonlinear behaviour of R_d necessitated solving a system of equations for accurate parameter extraction.

2. Circuit Diagram and Description

- The circuit under study consists of a variable DC power supply set to 24V, though the actual measured voltages vary due to load conditions.
- A resistor R1 is connected in series with a parallel combination of another resistor R_2 and an optocoupler diode (R_d).
- The optocoupler introduces a nonlinear resistance that changes with the current flowing through it, making the circuit's analysis more complex than a purely resistive network.
- The voltage across the parallel branch (R_2 || R_d) is measured and the total current through R1 is recorded for each test case.
- Since, the optocoupler's resistance is not constant, the effective resistance of the parallel combination varies with the applied current.
- This variability necessitates a systematic approach to derive the values of R_1, R_2 and R_d using multiple measurements to account for the nonlinear effects.

3. Schematic Representation



3.1 Key Observations

- The voltage across the circuit varies with current due to the nonlinear behaviour of R_d
- Multiple test cases provide voltage (V) and current (I) readings, allowing us to derive R_1, R_2 and R_d

3.2 Methodology

We apply Kirchhoff's Voltage Law (KVL) and Ohm's Law to the series–parallel network. The parallel relation used to express the node branch. Equations for each test case are set up and solved to estimate at the respective operating points

4. Derivation of Equations

- Using Kirchhoff's Voltage Law (KVL), the general equation for the circuit is,

$$\mathbf{V = IR_1 + I_{R2}R_2}$$

Where,

V	=	Supply Voltage
I	=	Total Current through R_1

I_{R2} = Current through R_2

R_1 & R_2 = Resistance

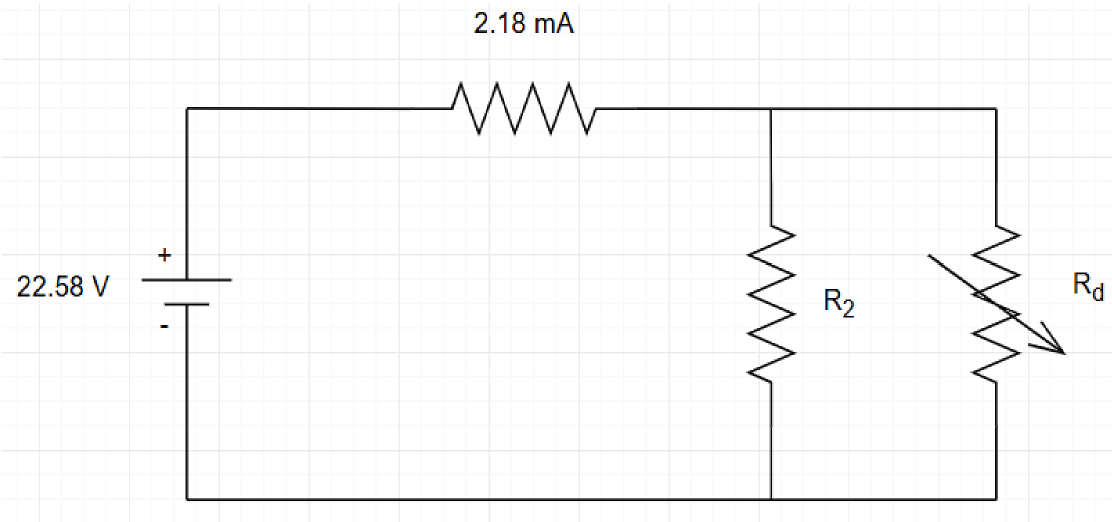
4.1 Experimental Measurements and Equation Formulation

- Three distinct test cases were conducted, each providing a unique set of voltage and current readings.
- These measurements are used to construct a system of equations that describes the circuit's behaviour under different conditions.

4.1.1 First Test Case:

Voltage = 22.58 V

Current = 2.18 mA



The equation derived is

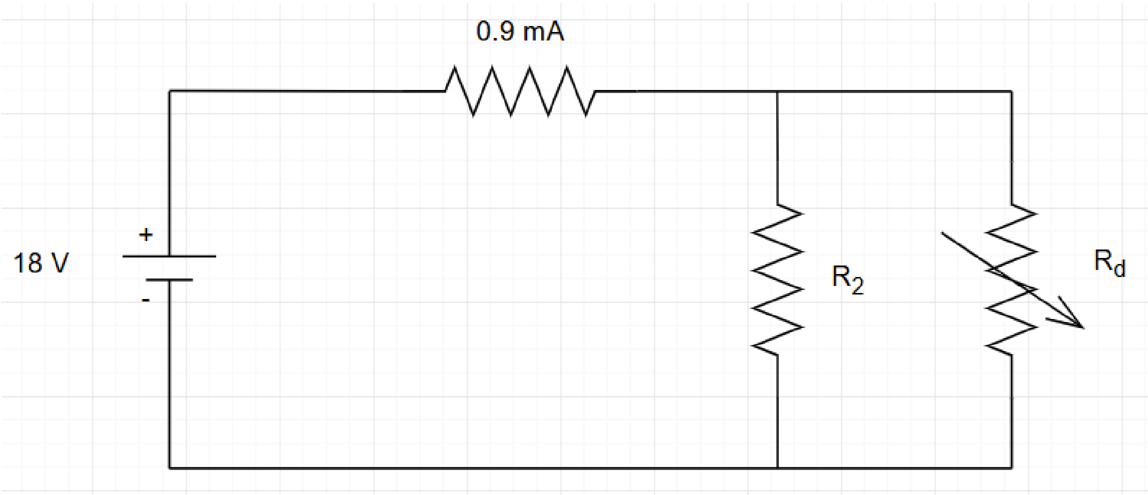
$$22.58 = 2.18R_1 + I_{R2}R_2 \quad \text{.......................................} \ (1)$$

Here, I_{R2} represents the current flowing through R_2.

4.1.2 Second Test Case:

Voltage = 18 V

Current = 0.9 mA



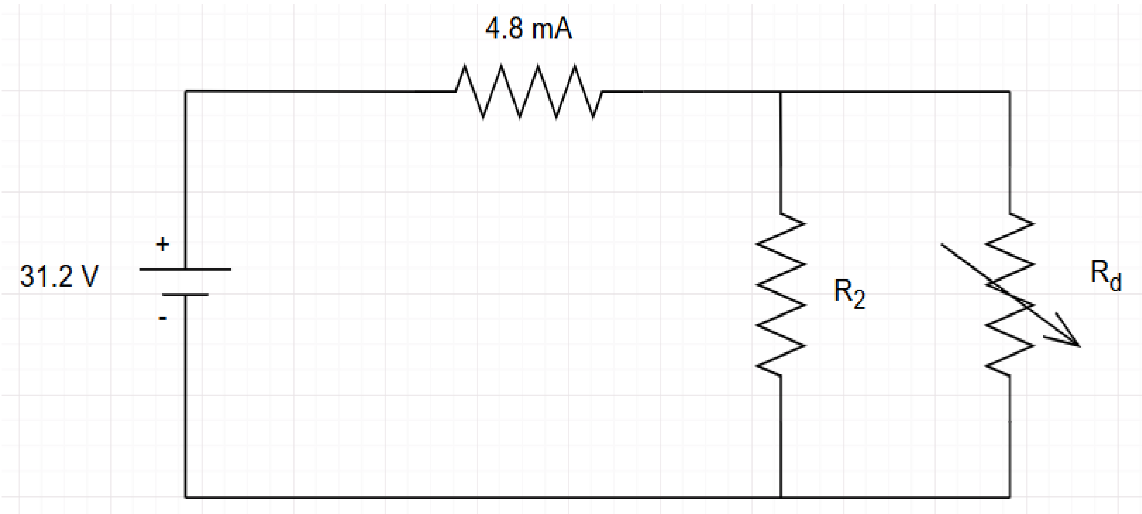
The corresponding equation is

$$18 \ = 0.9R_1 + I_{R2}R_2 \quad \text{......................................} \ (2)$$

4.1.3 Third Test Case:

Voltage = 31.2 V

Current = 4.8 mA



The equation becomes,

$$31.2 = 4.8R_1 + I_{R2}R_2 \quad \text{.....................................} \ (3)$$

These equations are solved simultaneously to isolate and determine the values of R1, R2 and Rd.

5. Solving for Circuit Parameters

Step 1: Determining $\mathbf{R_1}$

- By subtracting the second equation from the third, the term $I_{R2}R_2$ cancels out, allowing us to solve for R_1

$$31.2 - 18 = (4.8 - 0.9)\ R_1$$
$$13.20 = 3.9\ R_1$$
$$R_1 = \frac{13.20}{3.9} = 3.385\ k\Omega$$
$$\mathbf{R_1 = 3.385\ k\Omega}$$

Step 2: Calculating $\mathbf{I_{R2}R_2}$

- Substituting $R_1 = 3.385\ k\Omega$ back into the second equation yields

$$18 = (0.9)(3.385) + I_{R2}R_2$$

$$I_{R2}R_2 = 18 - 3.0465 = 14.95\ V$$
$$\mathbf{I_{R2}R_2 = 14.95\ V}$$

- Similarly, using the third equation

$$31.2 = (4.8)(3.385) + I_{R2}R_2$$
$$I_{R2}R_2 = 31.20 - 16.248 = 16.24\ V$$
$$\mathbf{I_{R2}R_2 = 16.24\ V}$$

Step 3: Resolving Discrepancies in $\mathbf{R_2}$

- The value of R_2 appears inconsistent when calculated from different test cases due to the influence of R_d. For instance,

From the second test case

$$\mathbf{R_2 = \frac{14.95}{0.9} = 16.61\ k\Omega}$$

From the third test case

$$\mathbf{R_2 = \frac{16.24}{4.8} = 3.383\ k\Omega}$$

This discrepancy arises because the effective resistance of the parallel combination $R_2 \,||\, R_d$ changes with current, highlighting the nonlinearity introduced by the optocoupler.

6. Detailed Analysis of Equivalent Resistance Equations

6.1 Understanding the Parallel Resistance Concept

The fundamental challenge in analysing this circuit lies in properly accounting for the parallel combination of resistor $\mathrm{R_2}$ and the optocoupler diode resistance $\mathrm{R_e}q$ is always less than either individual resistance.

This relationship is mathematically expressed through the standard parallel resistance formula:

$$\mathbf{R_e q = \frac{R_2\, R_d}{R_2 + R_d}}$$

In the circuit, this parallel combination forms the lower portion of a voltage divider, with $\mathrm{R_1}$ as the upper resistor

6.1.1 First Measurement Case Analysis

From the second experimental measurement, we obtained an equivalent parallel resistance of $16.61\ \mathrm{k\Omega}$. This given first key equation

$$\mathbf{16610 = \frac{R_2\, R_d}{R_2 + R_d}}$$

When we rearrange this equation to eliminate the denomination, we get

$$16610(\mathrm{R_2} + \mathrm{R_d}) = \mathrm{R_2\, R_d}$$

This represents a nonlinear relationship between R_2 *and* R_d*, making it impossible to solve for either variable independently. The equation tells us that the product of the resistances equals 16610 times their sum. This form is particularly useful because it preserves the fundamental relationship while preparing the equation for simultaneous solution with another measurement.*

6.1.2 Second Measurement Case Analysis

The third experimental measurement yielded a different equivalent resistance of $3.383\ \mathrm{k\Omega}$.

However, we must recognize that the optocoupler's resistance likely changed between measurements due to different operating currents.

To account for this, here introduce a scaling factor x *(calculated as 0.20 from the ration of the equivalent resistance)*

$$\boldsymbol{x} = \frac{\mathbf{3383}}{\mathbf{16610}}$$

$$\boldsymbol{x} = \mathbf{0.20} \quad \text{..} \ \mathbf{(4)}$$

That related the two operating conditions. This gives second key equation

$$\mathbf{3383} = \frac{\mathbf{R_2\ xR_d}}{\mathbf{R_2 + x\ R_d}}$$

Substituting, x = 0.20 and rearranging similarly gives

$$\mathbf{3383} = \frac{\mathbf{R_2\ 0.20\ R_d}}{\mathbf{R_2 + 0.20\ R_d}}$$

This equation now relates the same physical resistors but under different operating conditions, captured through the scaling of R_d

When we rearrange this equation to eliminate the denomination, we get

$$3383(\mathrm{R}_2 + 0.20\ \mathrm{R}_\mathrm{d}) = \mathrm{R}_2 0.20\ \mathrm{R}_\mathrm{d}$$

6.1.3 Establishing the System of Equations

Now, we have two properly formatted equations

$$\mathbf{16610(R_2 + R_d) = R_2\, R_d} \quad \text{.................................. (5)}$$

$$\mathbf{3383(R_2 + 0.20\ R_d) = R_2 0.20\ R_d} \quad \text{............................ (6)}$$

This system of nonlinear equations presents a mathematical challenge because the variables R_2 and R_d appear in both additive and multiplicative forms. The solution approach involves strategic manipulation to eliminate one variable at a time.

7. Equation Solving Methodology

To solve this system, we employ the following steps:

Step 1: Expand Both Equations

- Equation 5 becomes

$$\mathbf{16610\ R_2 + 16610\ R_d = R_2\, R_d} \quad \text{.......................... (7)}$$

- Equation 6 becomes

$$\mathbf{3383\ R_2 + 676.6\ R_d = R_2\ 0.20\ R_d} \quad \text{........................ (8)}$$

Step 2: Normalize the Equation

Here noticed that both equations contain the $R_2\, R_d$ term. To facilitate elimination, multiply the equations to make these terms compatible

- Multiply Equation 7 by 3383:

$$\mathbf{56191630\ R_2 + 56191630\ R_d = 3383\ R_2\, R_d} \quad \text{............... (9)}$$

- Multiply Equation 8 by 16610:

$$\mathbf{56191630\ R_2 + 11238326\ R_d = 3322\ R_2\, R_d} \quad \text{............... (10)}$$

Step 3: Subtract to Eliminate Terms

- Subtracting the modified Equation 9 from the modified equation 10

$$(56191630\ \mathrm{R_2} - 56191630\ \mathrm{R_2}) + (56191630\ \mathrm{R_d} - 11238326\ \mathrm{R_d})$$
$$= (3383\ \mathrm{R_2}\ \mathrm{R_d} - 3322\ \mathrm{R_2}\ \mathrm{R_d})$$

- This simplifies to,

$$\mathbf{44953304\ R_d\ = 61\ R_2\ R_d}$$

Step 4: Solve for $\mathbf{R_2}$

- Divide both sides by $\mathrm{R_d}$ (assuming $\mathrm{R_d} \neq 0$) to get

$$\mathbf{44953304\ = 61\ R_2}$$

Therefore:

$$\mathrm{R_2} = \frac{44953304}{61}$$

$$\mathbf{R_2 = 736939.41\ \Omega}$$

Step 5: Back – Substitute to Find $\mathbf{R_d}$

- Using Equation 8 with the known $\mathrm{R_2}$ value

$$(3383)(736939.41) + 676.6\ \mathrm{R_d}\ = (736939.41)(0.20)\ \mathrm{R_d}$$

Expanding:

$$2493066024.03 + 676.6\ \mathrm{R_d}\ = 147387.882\ \mathrm{R_d}$$

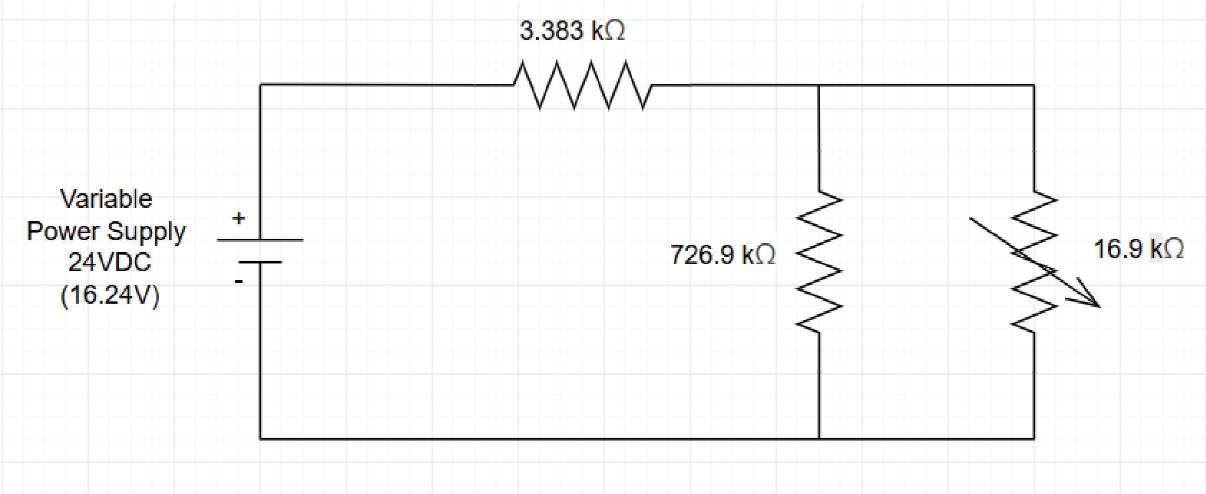
Rearranging:

$$2493066024.0\ = (147387.882 - 676.6)\ \mathrm{R_d}$$
$$2493066024.0\ = 146711.282\ \mathrm{R_d}$$

Solving:

$$\mathrm{R_d} = \frac{2493066024.03}{146711.282}$$

$$\mathbf{R_d = 16993.0082\ \Omega}$$



8. Results

Solving the system shows that Rd changes between cases, consistent with optocoupler behaviour. Small discrepancies in derived values between test cases are expected and reflect the operating-point dependence of Rd. Where needed, values can be linearized around a nominal current for design use.